LIFE INSURANCE

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Timetable – Day 1

- Actuarial Control Cycle
- Life insurance contracts – needs of consumers, risks and capital requirements – traditional contracts (with and without profits), unit-linked (investment) contracts, living benefits
- General business environment – distribution channels, and general affect of tax, regulation and professional guidance
- Risk – identification and resulting problems

Timetable – Day 2

- Models in life insurance – objectives and requirements, basic features
- Models in life insurance – valuing the liabilities and assets
- Data – requirements, checking
- Principles of setting supervisory reserves
- On-going solvency
- Fair value accounting and risk-based capital

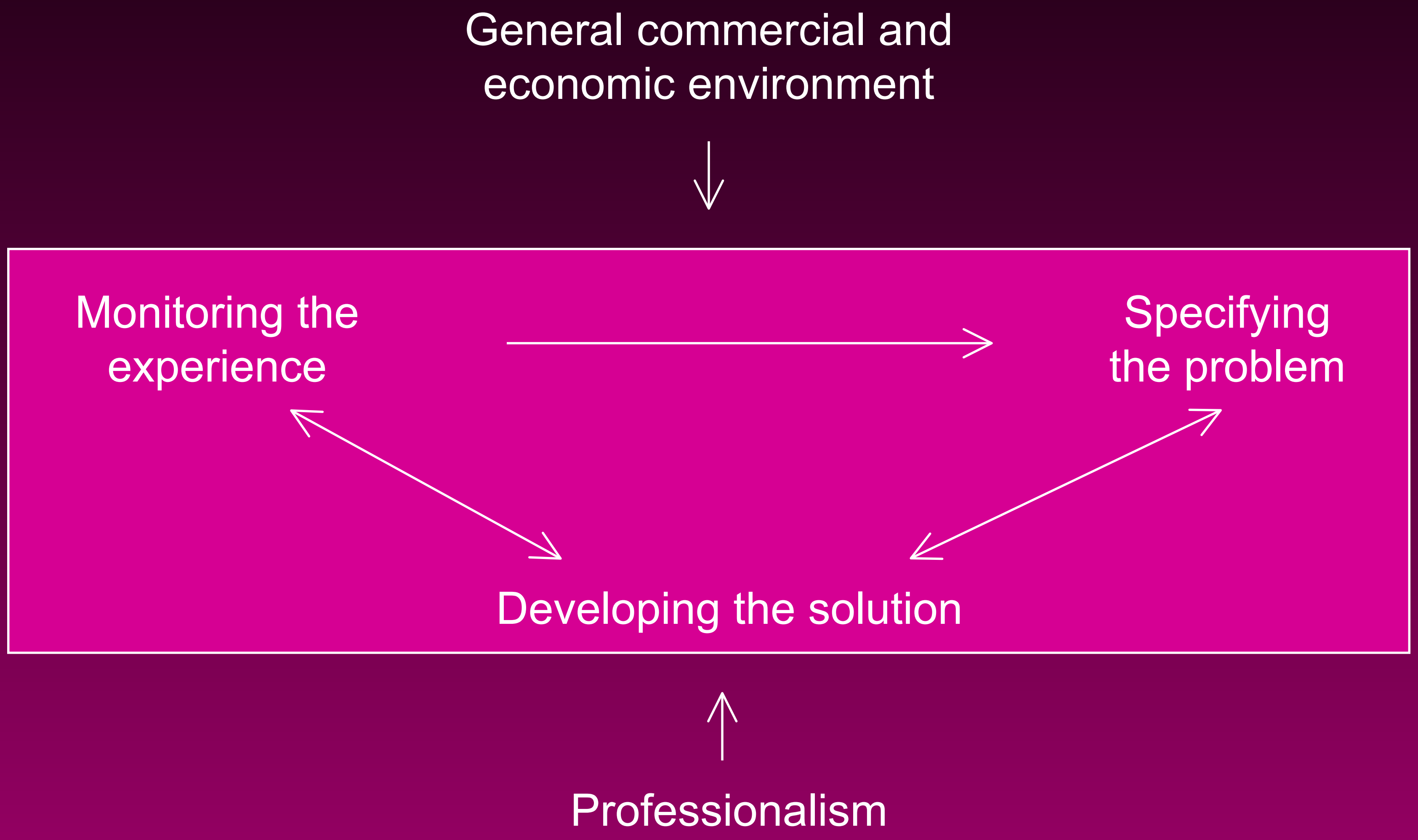
Timetable – Day 3

- Models in life insurance – embedded values
- Models in life insurance – product pricing
- Product design
- Product pricing assumptions + adequacy of premium rates + competition
- Reinsurance and underwriting

Timetable – Day 4

- Pricing and reserving for options and guarantees
- Discontinuance terms and alterations
- Investment and asset-liability matching
- Profit distribution, investment policy and management of risk
- Capital requirement
- Monitoring experience

Actuarial Control Cycle



Life insurance products

- Endowment assurances
- Fixed term contracts
- Whole life assurances
- Term assurances
- Convertible or renewable term
- Immediate annuity
- Deferred annuity
- Living benefits
- Unit-linked contracts
- Index-linked contracts
- Methods of distributing profit

Endowment assurances
consumer needs

- Cash benefit at a known future date
  - At retirement
  - To repay a loan
- Cash benefit on death
  - Protection for family
- Group contract (provided by employer)
  - Benefit at retirement and maybe on death

Endowment assurances
risks 1

- Cash benefit at future date →
  investment risk
  size depends on how we calculate benefit
- Cash benefit on death →
  mortality risk
  size depends on nature of benefit
  - significant benefit
  - return of premiums or “fund”
  - no death benefit

Endowment assurances
risks 2

- Expenses
- Withdrawals (surrenders)
- Capital requirement
  - design of contract
  - frequency of payment of premium
  - relationship between pricing and reserving bases
  - level of initial expenses

Fixed term contracts

- Consumer needs
  - certainty of payment at a known date
- Risks
  - investment, expense and withdrawal as for endowment
  - mortality – how long will premiums be paid
  - capital requirement – as for endowment

Whole life assurances

- Consumer needs
  - benefit on death – for funeral or to pay tax
- Risks
  - investment and mortality – depend on age at entry and duration in force
  - expenses – inflation risk at long durations
  - withdrawals and capital requirement – as for endowment

Term assurances
consumer needs

- Protects dependents against financial loss more cheaply than endowment etc
- Decreasing term assurance
  - to repay balance of loan on death
  - to provide income for children
- Key person insurance
- Group contract (provided by employer)
  - benefit on death of employee

Term assurances
risks

- Mortality
  - main risk
  - large anti-selection risk (except for group)
  - selective withdrawals
- Expenses – as for endowment
- Withdrawals – asset share can often be negative
- Capital requirement
  - basic requirement small
  - BUT increased by required solvency margin

Convertible/renewable term
consumer needs

- Attractions of a term assurance

  +

- Certainty of
  - conversion to an endowment/whole-life

    or
  - renewal when contract expires

Convertible/renewable term risks

- All those associated with a term assurance

  +

- Option gives rise to a significant anti-selection risk
- Higher reserve $\Rightarrow$ higher capital requirement

Immediate annuity contracts
consumer needs

- Can provide an income for remainder of life $\rightarrow$ after retirement
- or, for limited period $\rightarrow$ to pay school/university fees
- Income may be inflation-linked
- Group contract can provide pensions to employees after retirement

Immediate annuity contracts risks

- Main risk is a longevity risk – life lives too long
- Anti-selection risk – depends on whether life can choose to take out the contract
- Investment risk if cannot match
- Expense risk can be important as annuity may be paid for a long period of time
- Capital requirement can be significant – depends on
  - relationship between pricing and reserving basis
  - solvency margin requirements

Deferred annuity contracts
consumer needs

- Can use to build up an income after retirement
- May be option at retirement to take cash
- More flexible is endowment assurance (+ guaranteed annuity option)
- Group contract can fund pensions for employees after retirement

Deferred annuity contracts
risks

- Risks similar to those of an endowment combined with immediate annuity
- Additional investment and mortality risks if exists option to take cash
- Capital requirement is as for an endowment assurance.

Living benefits

- Long-term sickness contracts → provide an income if insured cannot work due to sickness or accident
- Long-term care contracts → provide financial security against risk of needing special care (usually after retirement)
- Critical illness contracts

Critical illness contracts
consumer needs

- Cash sum on diagnosis of a "critical illness"
- May be stand-alone contract
  - Usually require that life insured survives for at least say 30 days after insured event
- May be attached to another contract as a form of "accelerated" death benefit
  - Contract ends on payment of the benefit
  - Death is a "critical illness"
- May be additional benefit (e.g. attached to an endowment assurance)

Critical illnesses

- Cancer
- Heart attack
- Stroke – cerebrovascular incident
- Coronary artery bypass graft
- Multiple sclerosis
- Kidney failure
- Major organ transplant
- Total and permanent disability

Critical illness contracts
risks

- Main risk → critical illness incidence rates
- Improvements in diagnosis techniques
- Conflict with policyholder as to whether illness is a “critical” one
- Anti-selection at entry
- Capital requirement (under current EU rules) is not significant

Unit-linked contracts 1

- Consumer needs
  - premiums used to buy units which change in value according to the value of the “fund” of assets in which they are invested
  - specific deductions made to cover expenses and the cost of death, critical illness cover etc
  - higher expected benefit than under comparable non-linked contracts
  - greater flexibility in type and size of benefits and ability to vary premium

Description

Unit-linked contracts 2

- Risks
  - investment, mortality and expense risks usually carried by insured, if no guarantees
  - high marketing risk due to higher variance of benefit as compared with other comparable non-linked contracts
  - Capital requirement depends critically on the design of the contract

Index-linked contracts

- Consumer needs
  - consumer obtains a benefit that is guaranteed to move in line with the performance of a specified investment index
- Risks
  - main risk relates to investment – company may not be able to invest to match movements in the index
  - this contrasts with unit-linked where this risk is usually borne by the insured

Methods of distributing profit

- Additions to benefits (as used in UK)
- European methods
- Contribution method (as used in USA)

Additions to benefits

- Additions are made to the main benefit under the contract – during the course of the contract (<u>reversionary bonuses</u>) and at the end (<u>terminal bonuses</u>)
- The contract can be endowment, whole life or deferred annuity – with profit versions of other contracts are unusual
- A with profit version of unit-linked exists – <u>unitised with profits</u>.
- It can be quite difficult for policyholders to understand how the bonuses are determined.

Reversionary bonuses 1

- We can have a regular RB (given every year) or a special RB (given just once).
- Bonus can be calculated as
  - simple – a percentage of the initial benefit
  - compound – a percentage of the total current benefit
  - super-compound – as for compound, but with a different percentage applying to added bonuses than to initial benefit – first percentage is higher

Reversionary bonuses 2

- The different methods of calculation affect the speed at which profit is given to policyholders.
- They receive it quickest with a simple bonus and slowest with a super-compound bonus.
- This is important in terms of providing the company with an internal source of working capital.

Terminal bonuses

- Paid when contract terminates
- Amount determined when it is paid
- Allows for profit not yet given to policyholders by way of reversionary bonuses
- Amount may be expressed as
  - a percentage, possibly varying by duration, of total reversionary bonuses
  - a percentage of total existing benefit with percentage varying by duration in force
- Allows company greater freedom to invest in company shares.

Unitised with profits

- Have structure of a unit-linked contract
- Price of a unit is not based on value of assets in a unitised fund.
- Instead price is increased by bonus additions.
- Increases are regular plus terminal.
- May be a guaranteed level of regular increases.

European methods 1

- These aim to give profit to policyholders as it arises.
- Profit is divided by <u>type</u> for purpose of determining amount to give to policyholders.
- Often part of savings profit is given to policyholders and rest of profit to shareholders.

Types of profit

Savings profit

$$ {}_{t+1}V \frac{(i' - i)}{(1 + i)} $$

Expenses profit

$$ (GP - NP - E)(1 + i') $$

Mortality profit

$$ (S - {}_{t+1}V)[q_t \frac{1 + i'}{1 + i} - q_t'] $$

European methods 2

- Policyholder part may be expressed as a percentage addition to reserve (revalorisation method $\Rightarrow$ premium also increases) or as an addition to the benefit.
- Savings profit cannot be negative!
- This implies company should invest only in fixed interest securities.
- Easy for policyholders to understand how profit is distributed.

Contribution method

Profit given to each contract according to a dividend formula, traditionally of following form

$$(V_t + GP)(i' - i) + (q - q')(S - V_{t+1}) + [E(1+i) - E'(1+i')]$$

This amount (dividend) may be paid in cash or converted into an addition to the benefit.

General business environment

- Distribution channels
- Regulatory and fiscal regimes
- Professional guidance
  - Framework of points to consider by actuaries in order to maintain professional standards
  - Interpretation of Government Regulations

Distribution channels 1

- Insurance intermediaries (Brokers)
  - independent of any particular company
  - aim to find best contract for client
  - usually paid by insurance company
- Tied agents
  - insurance intermediaries “tied” to one life insurance company (e.g. employees of bank)
  - paid by insurance company

Distribution channels 2

- Own salesforce
  - employees of a life insurance company
  - paid by commission or salary or both
  - actively seek new clients
- Direct marketing
  - telephone selling
  - press advertising
  - internet

Distribution channels – effect 1

- Demographic profile of people insured
  - financially aware (rich) will go to brokers
  - ordinary people are sold contracts by own salesforce
  - there is class selection in terms of demographic experience
- Contract design
  - brokers can/have to sell complicated products
  - own salesforce require simple products
  - direct marketing requires very simple products

Distribution channels – effect 2

- Contract pricing
  - level of underwriting and hence mortality etc assumptions depend on sales channel
  - class selection also affects these assumptions
  - withdrawal assumptions affected by class selection and who initiated the sale
  - competition
    - brokers need competitive contracts
    - a bank must consider its good name
    - an own salesforce is not in a competitive situation

Regulatory regime 1

- Restrictions (to protect policyholders?)
  - types of contract
  - premium rates or charges
  - terms and conditions, e.g. surrender values
  - sales channels
  - minimum information to be given at time of sale
  - underwriting, e.g. use of genetic testing

Regulatory regime 2

- Further restrictions
  - amount of business a company can write through requirements for reserves and solvency margins
  - types of asset in which company can invest
  - amount of any particular type of asset
- Different regulatory regimes for institutions offering similar savings contracts, e.g. banks and insurance companies
- May be effects on contract design

Fiscal regime 1

- Life insurance business can be taxed in various ways
  - tax on profit
  - tax on investment income less (some) expenses
- Different types of business may be taxed differently – affects how contracts are written
- Different tax regimes for institutions offering similar savings contracts, e.g. banks and insurance companies

Fiscal regime 2

- Tax concessions for individuals may make certain contracts easier to sell
- Tax treatment of policy proceeds may distort buying habits
- May be effects on contract design

Risk

- <u>Sources of risk</u>
- <u>Problems to which the risks give rise</u>

Policy data

- Maintained by company
  - ⇒ actuary has no direct control over them.
- Risk that data will not be adequate, accurate and complete
  - ⇒ result of valuing liabilities incorrect.
- Sometimes need model of policy data
  - risk that model does not represent the data.

Other data

- For example data required to investigate mortality experience.
- Data may not be enough to provide reliable results.
- Data may be adequate but inappropriate.
- Any external data may be unreliable or inappropriate.

Mortality etc assumptions

- Three risks
  - "model risk" $\rightarrow$ model may not be appropriate
  - "parameter risk" $\rightarrow$ parameters for the model may not be appropriate
  - "random fluctuations risk" $\rightarrow$ actual experience different from expected even though model and parameters correct.

Mortality etc assumptions

- First two risks always exist
  - cannot predict the future
  - extent depends on reliability and applicability of data on which model and assumptions are based.
- Third risk arises because of
  - heterogeneity (lives are not the same)
  - law of large numbers unlikely to apply.

Investment performance

- Need assumptions with regard to returns on existing assets and assets to be bought in future
- Therefore can have both “model” and “parameter” risk – particularly if use stochastic assumptions.

Expenses

- Need assumptions for future inflation $\Rightarrow$ “parameter” risk and if use stochastic assumptions also a “model” risk.
- Total expense amounts from premiums received may not cover total expenses.

Withdrawals

- Have “parameter” risk and maybe also “model” risk.
- Also will be a “random fluctuations” risk
- Risk from selective effect of cancellations.
- Expense risk because of cancellations.

New business - nature and size of contracts

- Significant unintended change can change significantly risk profile or capital needs of business.
- Change can also cause expense problems.

New business - source of business

- Mortality and expense assumptions may prove incorrect due to unforeseen change in mix of business by source.
- May also apply to cancellation assumptions, if do not differentiate premiums.

Volume of new business

- Too much new business can lead to working capital being inadequate.
- Changes in volume of new business can invalidate expense assumptions

Guarantees and options

- Company offers terms in advance of the event to which they relate
  - guaranteed annuity option
  - option to renew a term assurance
- Actuary needs assumptions and a model to calculate the cost – both give rise to risk

Competition

- Competition may lead a company to take decisions that increase its risk profile, for example
  - reducing premiums
  - offering expensive guarantees and options
  - increasing “bonuses” to with profit contracts.

Management of the company

- Actuary may recommend actions to protect the company, but management does not have to follow them, for example
  - for competitive reasons
  - so as to maximise size of company
  - so as to maximise shareholder earnings

Operational risks

- Fraud
- Mis-selling
- IT issues
- Systems and control failures
- Management failures

Counterparties

- Reinsurance companies may not be able to meet their obligations to the life insurance company
- Counterparties, e.g. banks, may default under derivative contracts

Problems 1

- Management of a life insurance company will want to
  - maximise the profits of a company
  - maximise the return the company makes on its capital
- Usually these go together, but second is now more important

Problems 2

- Actuary should aim to assist company to meet its aims, in the context of
  - the company’s risk profile
  - the resources (working capital) available
  - the public interest

Problems 3

- Actuary must solve following problems
  - are data complete and accurate?
  - what contracts should the company offer?
  - what is the expected profit and its variance from selling a given product on given terms?
  - will the company have the resources to sell contracts on those terms?

Problems 4

- what is return on capital from developing and selling a new contract?
- what is expected profit and its variance from the existing business?
- what is effect on profit from particular discontinuance terms?
- what should be level of supervisory reserves?

Problems 5

- how should the company invest its assets so as to maximise expected return?
- will the company be able to achieve its long-term plans given available capital?
- how can risk management techniques – reinsurance, underwriting and policyholder participation in profits – be used to manage risk?

End

Unitised contracts

- Categorised by two things
  - Transparency of the charges made to cover expenses, additional benefits, etc
  - Concept of a unitised fund
- Special type of contract is an index-linked contract where benefit is linked to some investment index
- Can also have contracts known as “unitised with profits”

Unitised fund

- A company will usually have more than one fund
- A fund represents a separately identified set of investments
- Fund is divided into a number of “units” of equal value
- Value of a unit obtained by dividing “value” of investments in fund by number of units

Unitised fund (continued)

- Value of a unit known as its “bid price”
- The bid price does not change because of
  - New money going into the fund
  - Money going out to pay benefits (or charges)
- The bid price does change because of
  - Receipt of investment income
  - Changes in the value of the investments
  - Charges related to the value of the fund

Unitised fund (still more)

- Bid price is used when units are cancelled to pay benefits or meet charges
- New money buys units using the “offer price”
- The offer price is the bid price increased by the “bid-offer” spread

Charging structure

- Policy charge
- Unit allocation rate
- Bid-offer spread
- Fund management charge
- Benefit charges
- Surrender penalties
- Actuarial funding/unit cancellation rates

Policy charge

- Charged per contract and per time period
- Deducted from premium (unusual)
- Deducted from "unit account"
  - Cancel number of units equal in value to the charge
  - Usually at end of time period (illogical but time period usually a month)
  - Often inflation linked

Unit allocation rate

- Charge is (1 – unit allocation rate) x premium
- Allocation rate may be level throughout the term of the contract
- It may be lower in an “initial period” at start of contract
- Allocation in “initial period” may be to “initial units”

Bid-offer spread

- Can be expressed in two ways
  - as a proportion k of the bid price, so that

    Offer price = $(1 + k) \times$ Bid price
  - as a proportion k of the offer price, so that

    Bid price = $(1 – k) \times$ Offer price
- If k is the same in both cases, the amount of the charge is higher in the second case

Fund management charge

- The only charge that affects the price of a unit
- Expressed as a percentage of the value of the unit fund at the end of the time period before any other deductions are made
- Level of charge may be different for different unit funds

Benefit charges

- Modern practice is to charge for benefits – on death or critical illness etc – by using explicit charges deducted from a contract’s “unit account”
- Examples will show how the charges are calculated

Surrender penalties

- Deductions made from the bid-price value of the units in the contract's unit account if the contract is cancelled and a surrender value is taken
- May be expressed in two ways
  - A monetary amount – reducing with time
  - A percentage of the value of the unit account – reducing with time

Actuarial funding/unit cancellation rates

- Used with “initial units” as a way of making contracts more profitable
- Will see examples when look at pricing of unitised contracts

Why are unitised contracts so popular?

- Policyholder can see what his or her premium is being used for.
- Policyholder knows that he or she will get the full benefit of investment performance.
- Unitised contracts typically invest in company shares etc which perform better than Government or company stocks.
- BUT

What do policyholders forget about OR are not told?

- If investment performance is bad the policyholder directly and fully suffers.
- Charges for expenses etc may not be guaranteed.
- Hence policyholder may receive much less than expected.
- Traditional contracts have guarantees.
- Do policyholders understand the relationship between risk and reward?

Models in life insurance 1

- Why have models?
  - The real world is complex (and large)
  - A model is a simplification of some aspect of that world ...
  - ... that enables relevant calculations to be made and hence appropriate decisions to be taken

Models in life insurance 2

- Objectives of a model in life insurance
  - Prime objective is to enable an actuary to give the management appropriate advice so that the company can be run in a sound financial way (and meet the needs of its owners).
  - The actuary will use models in his/her daily work (pricing, reserving etc) and to provide checks and controls on the business.

Models in life insurance 3

- Requirements of a model
  - A model must be valid, rigorous and adequately documented.
  - Any model points used must be chosen to reflect adequately the distribution of the business being modelled.
  - Parameters used in a model must allow for all the features of the business being modelled that could significantly affect the advice being given.

Models in life insurance 4

- Requirements of a model (continued)
  - The inputs to the parameter values should be appropriate to the business being modelled and take into account the special features of the company and the economic and business environment in which it operates.
  - The outputs from the model should be capable of independent verification for reasonableness and should be communicable to those to whom advice will be given.

Models in life insurance 5

- **BUT** a model must not become so complicated that either
  - the results become difficult to interpret and communicate

  or

  - the model itself becomes too long or expensive to run.

Models in life insurance 6

- Basic features of a model for projecting life insurance business
  - Model needs to allow for all the cash flows that may arise
    - depending on the premium and benefit structure of each contract
    - and any non-guaranteed benefits, e.g. surrender values

Models in life insurance 7

- Model also needs to allow, where relevant, for all the cash flows that may arise from any supervisory requirement to hold reserves and to maintain an adequate margin of solvency.
- The cash flows need to allow for any interactions, particularly if assets and liabilities are being modelled together.

Models in life insurance 8

- If the business includes health options, the model needs to allow for the potential cash flows from such options.
- The model needs to be able to make use of stochastic models (e.g. for investment returns) and simulation, if appropriate, in order to assess the impact of financial guarantees.

Models in life insurance 9

- We need to choose the time period for calculating the cash flows in the projection, remembering that
  - the more frequently the cash flows are calculated the more reliable the output from the model
  - the less frequently the cash flows are calculated the faster the model can be run and results obtained.

Models in life insurance 10

- What can we do with projection models?
  - We can price life insurance contracts.
  - We can calculate the return on capital being used – for the whole company or a particular line of business.
  - We can calculate reserves (fair value reserves and risk based capital).
  - We can assess the profitability of existing business (embedded value).

Models in life insurance 11

- Sensitivity analysis
  - The results obtained from the models discussed depend on
    - the choice of model points
    - the values assigned to the parameters of the model
  - We may need to test for model point error – although we should choose an adequate set of model points to start with
  - Mis-estimation of parameters can be investigated using sensitivity analysis.

Valuing the liabilities

- Net premium method
- Gross premium method
- Discounted cash flow method
- Earned asset share
- Group life contracts
- Critical illness benefits
- Unit-linked contracts
- Other valuation methods

Net premium method 1

NP reserve =

(Expected present value of future benefits) – (Expected present value of future <u>net</u> premiums)

$$= \quad S_t \, \bar{A}_{x+t:\overline{n-t|}} \quad - \quad NP \, \bar{a}_{x+t:\overline{n-t|}}$$

$$NP \quad = \quad \frac{S_0 \, \bar{A}_{x:\overline{n|}}}{\bar{a}_{x:\overline{n|}}}$$

Net premium method 2

- Easy to understand, calculate and audit $\rightarrow$ method traditionally favoured by insurance supervisors
- Method makes no explicit allowance for future expenses, but can be adjusted so that it does, by valuing a premium P^* where

$$P^* = \text{Min}\,\{NP, (1-k)P\}$$

and P is gross premium

Net premium method 3

- Method assumes that expenses are the same each year – in practice they are much higher in first year.
- This means that reserve is higher than it needs to be.
- We can correct for this by valuing a Zillmerised net premium ZNP where

$$ZNP = NP + \frac{I}{\bar{a}_{x:\overline{n|}}}$$

Net premium method 4

- No allowance is made at all for expenses if no premiums are being paid, e.g. single premium contracts, $\Rightarrow$ may need to hold additional reserves.
- No explicit allowance is made for future profits. There is implicit allowance in $P - NP$ and in any margins in assumptions.
- Reserves are relatively insensitive to changes in the assumptions.
- Method cannot be used to value unitised contracts.

Gross premium method 1

- Liabilities consist of
  - expected future benefits B_s
  - expected future expenses E_s
  - expected future premiums (negative liability) P_s
- If L_s is the "expected liability outgo" at time s,

$$L_s = B_s + E_s - P_s$$

- Gross premium reserve =

$$\sum_s L_s (1+i)^{-s} = \sum_s B_s (1+i)^{-s} + \sum_s E_s (1+i)^{-s} - \sum_s P_s (1+i)^{-s}$$

Gross premium method 2

- For an endowment contract we have reserve at time t is

$$S_t \,\overline{A}_{x+t:\overline{n-t}|} + E^c \,\overline{A}'_{x+t:\overline{n-t}|} + E^r \,\overline{a}'_{x+t:\overline{n-t}|} - P(1-E^p)\,\overline{a}_{x+t:\overline{n-t}|}$$

where E^c = expected expenses on termination (i')

E^r = expected expenses each year (i')

E^p = expenses as proportion of premium

P = gross premium

$i' = (i-f)(1+f)^{-1}$

f = expected future expense inflation

Gross premium method 3

- Future profits to be given to policyholders can be allowed for in the assumptions used according to the method of distribution of profit.
- Explicit allowance is made for future expenses in an appropriate way and for all contracts.
- Reserves are relatively sensitive to the assumptions used – in particular to the rate of investment return used.
- It is difficult for insurance supervisors to check that the allowances for future profit and expenses are adequate and that the investment return rate is suitable.

Discounted cash flow method 1

- Gross premium method only works if the amounts L_s are all positive or start negative and become positive.
- Suppose this is not the case. $\geq$
- Two methods of calculating DCF reserve

$$(1) \quad \text{Reserve} = \underset{r}{\text{Max}} \sum_{s=1}^{r} \frac{L_s}{(1+i)^s}$$

where maximum is over all possible values of r

Discounted cash flow method 2

(2) Identify positive value of L_s furthest in future. If that occurs at time r then company needs a reserve at time $r - 1$, $V_{r\text{-}1}$, equal to

$$L_r\,(1+i\,)^{-1}$$

reserve at time $r - 2$, $V_{r\text{-}2}$, is then

$$(V_{r\text{-}1} + L_{r\text{-}1}\,)(1+i\,)^{-1}$$

and so on back to V_0

If the reserve becomes negative it is set to 0 and the process is continued.

Discounted cash flow method 3

- DCF method has same properties as GP method, but can be used will all contracts.
- For contracts with "normal" cash flows the DCF method produces the same figure for the reserve as does the Gross Premium method. Example

Earned asset share

- Retrospective accumulation of
  - gross premiums paid to date
  - less actual expenses
  - less cost of benefits provided

  using actual investment return achieved.
- It can be used
  - to help determine surrender values
  - in some profit distribution systems to determine the profit to give to policyholders.

EAS – Investment return

- Should be actual return on assets in which contract is invested, but we do not know what these are usually.
- Notional allocation of actual assets – most accurate approximation, but most difficult.
- Theoretical allocation of assets – less accurate, but easier.
- Use average return on total assets – even less accurate, but easy.

EAS – Expenses and benefits

- Should allow for actual expenses, but we will not usually know what these are.
- We can approximate using results of any company expense analysis.
- Cost of benefits can be approximated by multiplying benefit by an average rate of mortality etc.
- Such costs are not usually significant.

Group life contracts

- Special characteristics
  - contracts usually costed yearly
  - may be premium rate guarantee or with profit
- In theory need
  - unexpired premium reserve
  - incurred but not reported reserve
  - deficiency reserve (if have rate guarantee)
  - reserve for experience refund (if with profit)
- In practice may reserve 1 year's premium

Critical illness benefits

- Benefit is “stand-alone” or “accelerated”
  - value as if it were a benefit on death.
  - may also need an IBNR reserve
- Contract continues after payment of critical illness benefit
  - need to use discounted cash flow method
  - or maybe some approximate method

Unit-linked contracts

- Company’s liability defined partly in terms of units and partly in terms of currency.
- Matching requirements imply that we need to value each part separately to get
  - unit reserve
  - non-unit reserve (sterling reserve)

Unit-linked contracts – unit reserve

- Taken as the bid-price value of the units that the company should have allocated to contracts.
  - Units may be less than what the policyholders believe they have (explained later).
- If company mismatches (supervisors do not like this), also need a unit-mismatching reserve.

Unit-linked contracts – non-unit reserve

- Method needs to avoid future valuation strain.
- Possible discontinuities in cash flows mean that must use discounted cash flow method.
- Method applied by including all non-unit cash flows.
- Sum of unit and non-unit reserves must not be less than surrender value.
- Can have negative non-unit reserves.

Other valuation methods

- Traditional stochastic valuations
  - average over a range of deterministic valuations
  - DCF valuation using stochastic models and simulation
- Market consistent valuations
  - replicating portfolio
  - Stochastic market consistent techniques
    - closed form solutions – Black Scholes based formula
    - simulation using either “Risk neutral” or “Deflator” approach

Valuing the assets

- Book value
- Market value
- Discounted cash-flow value
- Value of assets for supervisory purposes

Book value 1

- Historic cost
- Ignores subsequent changes in value
  - ⇒ May overstate true value
  - ⇒ May understate true profit

Book value 2

- Readily ascertainable
- Stable value
- Adjusted book value
  - $\Rightarrow$ lower of book value and market value

Market value 1

- Amount the company will get if it sells asset on the open market

  ⇒ true value (market-based value or fair value)

- Realistic value

- Objective

Market value 2

- May not be available

  ⇒ subjective

- Can change a lot and quickly

- May be influenced by short-term considerations

Discounted cash-flow value 1

$$\sum_{t=1}^{\infty} \frac{A_t}{(1+i)^t}$$

A_t = expected income in year to t

i = rate of return for discounting

Discounted cash-flow value 2

- We need assumptions
  - $\Rightarrow$ subjective
- Gives a more realistic long-term value?
- Discount rate ignores risk
  - $\Rightarrow$ leads to inconsistencies
  - $\Rightarrow$ can lead to inappropriate investment decisions
- Complicated

Value of assets for supervisory purposes

- Choice is between book value and market value.
- Many insurance supervisors require book value, but allow amortisation of redeemable stocks, e.g. in most EU countries.
- Some supervisors allow market value, but require companies to assess affect on solvency of changes in market values, e.g. in the UK and Ireland.

Data

- Already covered under “Risks”
  - different types of data
  - why data needs to be complete and accurate
- Data requirements for a valuation
- Checks of policy data

Data requirements

- Type of contract
- Date of birth of life insured
- Sex of life insured
- Date the contract started
- Term of the contract
- Gross premium
- Premium payment term
- Frequency of payment of premium
- Current level of benefits provided

Checks of policy data 1

- Data reconciliation checks
  - group data, e.g. by contract type and year of entry
  - check that

{data at previous valuation}

+ {business on}

– {business off}

= {data at current valuation}

Checks of policy data 2

- We can use this to check
  - benefits
  - gross premium
  - units allocated to unit-linked contracts
- The systems used to produce “on” and “off” data must be checked regularly to ensure they are working properly.
- “On” and “off” data should be checked against any appropriate accounting data.

Checks of policy data 3

- Consistency checks – for example
  - average benefit or premium for each group
  - ratio of benefit to premium for each group

  should be sensible and consistent with previous year.
- Unusual values checks
- Analysis of surplus/profit

Setting supervisory reserves

Some actuarial principles

Actuarial principles

- All liabilities can be met.
- Prudent valuation of all future liabilities.
  - guaranteed benefits ($\Rightarrow$ reserve not negative)
  - bonuses already guaranteed
  - options available
  - future bonuses of all kinds
  - expenses
  - premiums

More principles

- Prudent valuation is not a best estimate valuation.
  - $\Rightarrow$ include margins for adverse deviations
- Take account of nature, term and method of valuation of the assets.
- Can use approximations or generalisations.

and more

- Rate of interest is prudent and based on
  - currency
  - returns on existing assets
  - expected return on future investments
- Mortality assumption is prudent and based on
  - type of contract
  - country where life insured lives

and more

- Expenses are prudent based on
  - type of contract
  - expected administrative costs
  - expected commission costs
- If no explicit reserve for future bonuses must use a lower interest rate than that determined earlier.

and more

- Recognise profit in an appropriate way.
- Value of liabilities should not be subject to discontinuities.
- The company should disclose the methods and assumptions that it uses to value the liabilities.

Some additional principles

- Avoid future valuation strain.
- Can use a Zillmer adjustment subject to limits.
- Allowance for expenses not less than that needed if the company closed to new business.
- No allowance for cancellations if reduces reserve.

Reserving versus pricing assumptions 1

- In some countries (many European countries for example) it is standard practice to price contracts using prudent assumptions and to use same assumptions for supervisory purposes.
- This is particularly suitable where contracts are traditional European-style with profit.
- Approach is not justifiable for other types of contract.

Reserving versus pricing assumptions 2

- In other countries (for example UK and most other English-speaking countries) it is standard practice to price contracts using assumptions that reflect future expected experience, with risks to the company being allowed for through risk discount rate.
- In this case, it would not be appropriate to use the same assumptions for pricing and for reserving.

Sensitivity analysis

- Assumptions for reserving need to be prudent estimates of future experience.
- Prudent $\Rightarrow$ expected values + margins for adverse future experience.
- Assess these margins using sensitivity analysis.
- Can also use sensitivity analysis to determine the need and extent of any global reserves to cover potential future adverse experience.

Solvency requirements 1

- Usual for insurance supervisors to require that companies maintain a specified level of “solvency capital” in addition to technical reserves
  - to provide an additional level of security for policyholders
  - o provide trigger points for intervention before actual insolvency occurs.

Solvency requirements 2

- Adequacy of technical reserves should be considered together with nature of required “solvency capital”.
- Some countries have relatively weak reserves, but strong requirements for solvency capital (Canada).
- Others have strong reserves and weak solvency capital (EU countries).
- EU is currently considering changing its approach (Solvency II discussions)

Required solvency margin – EU

- 4% of gross mathematical reserve
  - reduced to allow for reinsurance subject to maximum reduction of 15%
- 0,3% of gross capital at risk
  - lower percentage applies if term of contract 5 years or less
  - reduced to allow for reinsurance subject to a maximum reduction of 50%

On-going solvency 1

- Why project solvency?
  - Supervisory authorities usually only require a static test of solvency.
  - This will not show whether the company can withstand future changes in
    - the external economic environment
    - the particular experience of the company

On-going solvency 2

- What we need is a dynamic assessment of solvency – dynamic solvency testing.
- This involves a projection forward of the revenue account and balance sheet for a sufficient period of years so that the full effect of any potential risks become apparent.

On-going solvency 3

- Alternative methods of assessing solvency
  - use of the supervisory value of assets and liabilities
  - use of expected value of liabilities with market value of liabilities
- The first gives the stronger test.

On-going solvency 4

- For dynamic solvency testing also need a basis for doing projections
  - deterministically using expected assumptions together with a number of sets of assumptions with margins (scenarios)
  - stochastically, i.e. stochastic assumptions and simulation, $\rightarrow$ "probability of ruin".
- Also need to decide whether to include expected future new business.

Fair value accounting

- International Accounting Standards Board is proposing that the accounts of insurance companies should show assets and liabilities at **fair values**.
- Fair value defined as

  “the amount by which an asset could be exchanged or a liability settled between knowledgeable, willing parties in an arm’s length transaction”.

Fair value of assets

- If the assets are traded in a deep, liquid market, fair value = market value.
- Otherwise, fair value is a calculated value which is consistent with the market values of traded securities.

Fair valuation of liabilities 1

- If the liabilities are traded in a deep, liquid market, fair value = market value.
- Most liabilities are not traded.
  - Calculated value using assumptions, risk provisions and discount rates that an independent market-place participant would make to take on the liability.

Fair valuation of liabilities 2

- Where possible, assumptions etc. would be calibrated to similar quoted instruments.
- Liability cash flows would include a margin over best estimate assumptions (Market value margin) to allow for non-diversifiable risks.
- Liability valuations would not in general depend on how the corresponding assets were invested, except in the case of contracts where benefits depend on the choice of investments.

Fair valuation of liabilities 3

- These proposals have met with a lot of opposition, particularly from insurance supervisors in Europe and America.
- But adoption of the proposals might be an opportunity to build a consistent system of reporting for prudential supervisory purposes.

Accounting standard – current position

- Assets valued at market value, or a value that is consistent with market value.
- Liabilities valued according to local GAAP.
  - In most of Europe $\Rightarrow$ net premium reserve calculated on premium basis assumptions.
  - Not consistent with market value of assets.

Prudential reporting 1

- Six principles (British)
  - Margins in fair value liabilities are not adequate for prudential supervisory purposes – further margins should be held.
  - These additional margins should be transparent so as to judge relative solvency of companies.
  - System should ensure a level playing field between different institutions offering similar products.

Prudential reporting 2

- Six principles (contd.)
  - System should reinforce good risk management practice – good practice rewarded by lower capital requirements.
  - Capital requirements should have regard to assets backing the liabilities and policyholder options.
  - Should be a set of trigger points to give early warning of potential insolvency.

Prudential reporting 3

- Implementing these principles
  - General purpose accounts would define the capital available to the company.
  - Prudential supervisory standards determine the minimum amount of capital required.
  - This minimum amount of capital should be determined using a risk-based capital approach – capital requirement related to risk of default or ruin and calculated in a (more or less) scientific way.

Calculation of solvency margin requirements 1

- Formula method – as used in EU
  - Simple and easy to audit
  - Produces arbitrary results
  - Focuses on only certain types of risk
  - Does not give credit to companies who set up prudent reserves.
  - Not sensitive to individual risk profile of a company.
  - Does not take credit for any risk management process.
  - Not explicitly dynamic (forward-looking).

Calculation of solvency margin requirements 2

- Risk-based capital
  - Calculate capital requirements which reflect the size and overall risk exposures of an insurer.
  - Similar to formula method in that sub-results are established by applying factors to exposure proxies, e.g. asset risk, underwriting risk etc.
  - But uses more risk proxies and factors than the formula method and these are combined with a more sophisticated mathematical formula.

Calculation of solvency margin requirements 3

- Risk-based capital
  - Better than formula method.
  - Not simple to apply, but does not need complex systems and model to calculate results.
  - Calculation based on factual and historical data $\Rightarrow$ no subjectivity.
  - Capital requirements still to some extent arbitrary.
  - Does not cover all risks or interactions between them.
  - Not dynamic.

Calculation of solvency margin requirements 4

- Scenario-based approaches
  - Attempt to analyse the impact of specific risk variables to company specific exposure.
  - Capital requirements based on the worst-case outcome from a set of scenarios applied to the insurance company’s financial model.
  - Financial model is typically dynamic (but could be static) and produces deterministic cash-flow and balance sheet projections.
  - Scenarios will usually consist of various sets of future inflation and interest rates, returns on assets etc.

Calculation of solvency margin requirements 5

- Scenario-based approaches
  - Allows for a straightforward and intuitive interpretation of results.
  - Capital requirements more clearly defined.
  - Scenarios provide flexibility in the scope of risks considered.
  - Framework for considering risk-interaction.
  - Can be dynamic and forward looking.
  - Result dependent on the specific set of scenarios used.
  - Models can be complex and more sophisticated versions impose considerable data requirements.

Calculation of solvency margin requirements 6

- Probabilistic approaches
  - Attempt to cover the full range of risk variables which are sampled from statistical distributions using simulation.
  - Therefore consider a wider range of outcomes, the likelihood of adverse development and the interaction of risk variables.
  - Give the full probability distribution of possible outcomes.
  - Capital requirements calculated from features of the distribution using ruin-probability or expected policyholder approaches (or other risk measures).

Calculation of solvency margin requirements 7

- Probabilistic approaches
  - Greater flexibility.
  - Specifically covers interaction of risks.
  - Attempts to combine and refine distinct risk categories.
  - Results much more difficult to understand.
  - Most complex approach with significant data requirements.
  - Where data not available can be quite subjective.
  - Difficult to codify or standardise.

Solvency II

- Possible future in EU countries
  - 3-pillar approach
  - Classification of risks
    - Risks at company level
    - Risks faced by insurance industry (systematic risks)
    - Risks faced by the economy (systematic risks)

3-pillar approach

Pillar I	Pillar II	Pillar III
Financial resources	Supervisory review	Market discipline
Minimum capital requirements set for firms generally using a risk based approach assessed by reference to underwriting information, and assets and liabilities in the financial statements. Options for firms to graduate to scenario approaches and internal (probabilistic) models. Group solvency requirements taking account of additional risks at group level. Other prudential rules (assets and liabilities).	Assessment of the strength and effectiveness of risk management systems and internal controls including review of: - exposures (including the reinsurance programme); - internal risk models; - stress testing of technical provisions and assets; - fitness and propriety of senior management; - asset/liability mismatch.	Disclosures recommendations and requirements create transparency by allowing market participants to assess key information on scope of application, capital, risk exposures, risk assessment and management processes, and capital adequacy of the insurance undertakings. Disclosures on risks: - risks; - key sensitivities and scenario analysis on assets and technical provisions.

Risks at company level

- Pure underwriting
- Underwriting management
- Credit
- Reinsurance
- Operational
- Investment
- Liquidity
- Matching
- Expenses
- Lapses
- Provisioning

Risks faced by insurance industry

- Jurisdictional and legal
- Market changes

Risks faced by the economy

- Market value fluctuation of investments
- Environmental changes
- Social/political changes
- Economic cycle
- Inflation rate
- Interest rate
- Exchange rate
- Technological changes

Embedded values

- Embedded value = present value of the expected future profits from the existing business.
- Why calculate it?
  - To put a value on the company for sale purposes.
  - Change in value gives a more realistic measure of profit.
  - To allow management to monitor company against pricing bases and business plan assumptions.
  - As part of an incentive scheme for senior staff.

Calculation of embedded value

- Construct model of existing business.
- Project forward all cash-flows using set of (realistic) assumptions.
- Project supervisory value of liabilities.
- Calculate expected (shareholder) profit each year = total net cash flow in year less increase in value of liabilities.
- Discount the expected profit figures at a risk discount rate.

Example

Analysis of change in embedded value

- Why analyse the change?
  - check of the embedded value calculation
  - comparison of actual against expected experience helps in revising assumptions (control cycle)
  - provides management with value of new business written
  - identification of sources of loss so that action can be taken to limit future losses
  - identification of unprofitable contracts so that they can be redesigned or re-priced.

Goodwill value 1

- GV = value of expected profits from expected future new business
- Project into future expected new business – say NB_t in year t
- At end of each year t calculate value of expected profits from NB_t, using embedded value method, to get $EV(NB_t)$

Goodwill value 2

- Discount these expected profits at a suitable rate of return, say j, to get

$$GV = \sum_{t=1}^{\infty} \frac{EV(NB_t)}{(1+j)^t}$$

- Simplification is to assume that

  $$NB_t = NB_0 \times (1+g)^t$$

  for some g to get

$$GV = EV(NB_0) \sum_{t=1}^{\infty} \left(\frac{1+g}{1+j} \right)^t$$

$$\cong \frac{EV(NB_0)}{j-g}$$

Product pricing

- Equation of value method
- Emerging costs method
- Pricing group life contracts
- Pricing critical illness
- Pricing unit-linked contracts Example

Equation of value method

EPV – gross premiums $= \quad P\sum_{t=0}^{n-1} {}_t p_x v^t \quad = \quad P\ddot{a}_{x:\overline{n}|}$

EPV – expenses and commission $= \quad \alpha B + \beta P\ddot{a}_{x:\overline{n}|} + \gamma B\ddot{a}_{x:\overline{n}|}$

EPV – benefits $= \quad B\left(\sum_{t=0}^{n-1} {}_{t|}q_x v^{t+½} + {}_n p_x v^n\right)$

$= \quad B\bar{A}_{x:\overline{n}|}$

Equation of value - European

$$P = \frac{B\left(\bar{A}_{x:\overline{n}|} + \alpha + \gamma\, \ddot{a}_{x:\overline{n}|}\right)}{(1-\beta)\, \ddot{a}_{x:\overline{n}|}}$$

Equation of value - Britain

$$P = \frac{B\bar{A}_{x:\overline{n}|} + E_i + E_r(\ddot{a}_{x:\overline{n}|} - 1) + E_c\bar{A}'_{x:\overline{n}|}}{(1 - c_r)\ddot{a}_{x:\overline{n}|} - (c_i - c_r)}$$

Immediate annuity - Britain

$$P = \frac{12An\, a_x^{(12)} + E_i + E_p\, a_x'^{(12)}}{1 - c_i}$$

Equation of value method

- Simple
- Works well for with profit contracts.
- Not good otherwise
  - Does not quantify profit.
  - Ignores reserving requirement.
  - Difficult to allow for cancellations.
  - Very difficult to use with unit-linked contracts.

Emerging costs method 1

- Aim of method is to answer question – what premium do we charge for a contract so that expected (shareholder) profit is acceptable?
- Also answers questions
  - What is the sensitivity of that profit to changes in future experience?
  - What are the capital needs of the contract?

Emerging costs method 2

- Project forward all the cash flows, allowing for all possible benefits under the contract.
- Calculate at each future point in time a “profit” figure.
- Discount the separate “profit” figures at a suitable rate of return (risk discount rate) $\rightarrow$ total expected profit at start of the contract.
- Assess acceptability of this total against profit requirement.

Example

ECM – risk discount rate 1

- Risk discount rate = rate of return required by shareholders.
- This rate takes account of
  - Risk-free rate of return in the investment market
  - Degree of risk associated with the profit stream from the contract.

ECM – risk discount rate 2

- Allowance for risk depends on
  - the margins, if any, in the assumptions used to project the cash flows
  - the degree of confidence the company has in the assumptions
  - the level of guarantees in the contract
  - the existence of any policyholder options
  - the extent, if any, to which the risk may be hedged.

ECM – profit criteria

- Management want some means of deciding how profitable a contract is.
- Three methods
  - <u>Net present value</u> (total discounted profit)
  - <u>Rate of return on capital</u>
  - <u>Discounted payback period</u>

ECM – Net present value

- best criterion according to economic theory
- assumes use appropriate risk discount rate
- does not allow for law of diminishing returns
- ignores competition
- usual to express as a percentage of initial commission
  - commission reflects work required to sell contract
  - equates shareholders’ and sellers’ interests
- may express as percentage of value of future premiums – profit margin

ECM – Rate of return on capital

- Also known as the “Internal rate of return”
- Represents the rate of return, if it exists, which makes the total discounted profit equal to zero
- Not entirely suitable as a profit criterion as it ignores the risk profile of the profit stream.
- Modern development is to calculate the “risk adjusted rate of return”, i.e. rate of return after allowing for risk in the profit figures.
- In theory rate of return may not exist, but in a perfect market this should not happen.

ECM – Discounted payback period

- The policy duration at which the profits to date first have a discounted value of at least zero.
- It indicates how long it takes the company to get back the capital invested in the contract allowing for interest at the risk discount rate.
- The shorter this period the quicker the company can recycle its available capital → important for a company with limited capital.

Example

ECM - Advantages

- Complicated
- Can price any contract
- Allows for reserves and cancellations
- Quantifies the expected profit from the contract
- Can investigate the sensitivity of the expected profit

Sensitivity – Investment return

	With profit	Non-profit
6%	17%	-109%
8%	33%	-30%
10%	50%	50%
12%	69%	129%
14%	90%	209%

Sensitivity – Inflation rate

	With profit	Non-profit
4%	54%	58%
6%	52%	54%
8%	50%	50%
10%	48%	45%
12%	45%	40%

Sensitivity – Investment/Inflation rates

	With profit	Non-profit
6%/4%	21%	-101%
8%/6%	35%	-25%
10%/8%	50%	50%
12%/10%	67%	125%
14%/12%	85%	200%

Sensitivity – Mortality rates

	With profit	Non-profit
0,5	52%	53%
0,75	51%	52%
1	50%	50%
1,5	48%	46%
2	46%	43%

Sensitivity – Cancellation rates

	With profit	Non-profit
0,5	56%	62%
0,75	53%	56%
1	50%	50%
1,5	45%	39%
2	40%	29%

Group life contracts

- Priced on a yearly risk-premium basis.
- Premium each year =

  benefit $\times$ mortality rate + expenses
- Calculation may be done for each life and then summed.
- If group is big may use benefit for whole group with an average mortality rate.
- Premiums may be guaranteed, but this has solvency margin implications under EU rules.
- With profit version also exists.

Critical illness

- Stand-alone benefit – price like a term assurance.
- Accelerated death benefit – price as basic contract with transitions the sum of critical illness and “death” rates.
- Otherwise – probably have to use emerging costs method Example

Product design 1

- Contracts should be profitable
- Contracts should be marketable
- Contracts should be competitive
  - may go against contracts being “profitable”
- Minimise financing needs
  - easier to do with unit-linked contracts
  - can make contract less marketable

Product design 2

- Risk characteristics of contract - are they acceptable?
- Are any guarantees or options too onerous?
  - guarantees and options increase marketability
- Minimise sensitivity of profit
  - easier to do with unit-linked contracts
  - guarantees and options increase sensitivity
  - BUT increase marketability

Product design 3

- Extent of cross-subsidies, for example between large and small contracts
- Administration systems needed.

Product pricing assumptions

- Principles of setting assumptions
- Adequacy of premium rates
- Competition

Assumptions - mortality

- Represents expected future experience.
- Base on past experience
  - of company if exist, relevant and credible
  - maybe from reinsurance company.
- Take account of future trends if appropriate.
- Degree of prudence depends on extent of mortality guarantee.

Assumptions - critical illness

- Represents expected future experience.
- Based on past experience
  - of company if exist and credible (unlikely)
  - statistics produced by Health Ministry
  - from reinsurance company.
- Take account of future trends if appropriate.
- Prudence depends on guarantee given.

Assumptions - Expenses

- Represent expected future expenses
  - express as current expenses + inflation rate
- Base on investigation of recent expenses.
- Margin for prudence in inflation rate depends on expense guarantee given.
- Need to decide how to allow for expenses that do not vary by the size of the benefit (or premium).

Expenses independent of contract size

- Individual calculation of premium rates
- Policy fee addition to a premium that "ignores" these expenses
- "Sum assured differential" method
  - charge different premium rates by bands of benefit

Assumptions - Cancellations

- Represent expected future cancellation rates.
- Base on past experience.
- BUT future experience may be significantly different due to
  - changes in sales channel
  - target market
  - changes in economic conditions.

Assumptions - Investment return

- Represents expected future return, based on
  - extent of investment guarantees given
  - significance of the assumption
  - extent of (unhedgable) reinvestment risk
  - intended investment policy
  - current investment returns

Assumptions - Unit growth rate

- Represents expected return on the investments in each unitised fund.
- Should be realistic as investment risk is borne by the policyholder.

Assumptions - Group life 1

- Expenses as before.
- Investment return not important as one year contracts.
- Mortality assumptions based on
  - occupations involved
  - geographical location
  - free cover limit

Assumptions - Group life 2

- Mortality assumptions also based on
  - past scheme experience, but need to consider its credibility
  - whether membership is voluntary
  - extent of any premium rate guarantee given.

Adequacy of premium rates 1

- What are “adequate premium rates”?
  - Premiums which on reasonable assumptions are such that they repay any initial capital required (at the risk discount rate?).
- Is it a problem to sell contracts with “inadequate premiums”?
  - In theory NO, provided that the company has enough capital to set up an appropriate mathematical reserve at the start of the contract.

Adequacy of premium rates 2

- And in practice?
  - Strict limits would need to be set on amount of such contracts that can be written.
  - This could be a problem as contracts will probably be very competitive and therefore easier to sell than other contracts.
  - Insurance supervisors are unlikely to be happy, but prior approval of rates not required.
  - Role of Actuary?

Competition - what to do

- Product differentiation
- Remove expensive guarantees or options
- Sell to a different target market
- Change sales channel
- Use “marginal expense costing”
- Persuade shareholders to accept lower profit
- Abandon the project

Competition - what not to do

Do not change actuarial assumptions

Managing risk - reinsurance

- Why reinsure?
- Main types of reinsurance contract
- Specifying the amount to reinsure
- Facultative and treaty
- How much to reinsure – retention limits

Why reinsure?

- Claim payout fluctuations
  - reduce variance of expected mortality etc experience
  - variance can be high due to
    - small number of contracts with high level of cover $\rightarrow$ use original terms or risk premium reinsurance
    - lives are not independent risks $\rightarrow$ use catastrophe reinsurance

Why reinsure? contd.

- New business
  - reduce financial risk from new business by
    - increasing available capital
    - reducing financing requirement
  - original terms reinsurance or some form of financing reinsurance can be used
- Technical assistance
  - underwriting, product design and pricing, and systems design
  - particularly useful for new company or new line of business

Main types of reinsurance contract

- Original terms (coinsurance) reinsurance
- Risk premium reinsurance
- Catastrophe reinsurance
- Financing reinsurance

Original terms (coinsurance) reinsurance

- Share all aspects of contract with reinsurer
- Reinsurer pays commission based on
  - ceding companies premium rates
  - Likely future experience
  - knowledge of quality of underwriting
  - its own need to make a profit
- This commission covers part of original commission and part or all of other expenses
- may have “deposit back” of reserves

Risk premium reinsurance

- Reinsure only part of sum at risk
- Two approaches to calculating sum at risk → covered later
- Reinsurer's premium rates based on
  - likely future experience of business
  - plus additions for expenses and profit
- Reinsurer may or may not guarantee these rates
- Method can also be used for critical illness etc. benefits

Catastrophe reinsurance

- Aim is to reduce risk due to non-independence of risks
- Cover only available on yearly basis
- Reinsurer pays if a "catastrophe" occurs
  - minimum number of deaths – say 5
  - from a single event
  - within a specified time – say 48 hours
- Contract will specify how much reinsurer pays
- Cover excludes – war risks, epidemics and nuclear risks

Financing reinsurance

- Risk premium reinsurance + “loan”
  - loan expressed as reinsurance commission to avoid needing to set up a reserve
  - repayments spread over a number of years and added to reinsurance premiums
- Loan against future profits
  - contingent loan and hence again no reserve needed
  - can be used by an insurance company to improve its solvency position or to fund a new project

Specifying the amount to reinsure

- Original terms method
  - Individual surplus – excess of original benefit over company's retention limit
  - Quota share – percentage of each policy
  - May be a mixture of both
- Risk premium method
  - Constant retention – excess of sum at risk above a specified amount
  - Reducing retention – specified percentage of sum at risk

Facultative and treaty

- "Facultative" = may
- "Obligatory" = has to
- Hence we have (insurer/reinsurer)
  - facultative/facultative
  - facultative/obligatory
  - obligatory/obligatory
- Last two types are usually formalised in a "Treaty" – the first may be.

Retention limits 1

- In theory need to estimate the statistical distribution of the mortality etc costs of the portfolio on various assumed retention limits.
- Then choose retention so that variance of cost is below a specified (low) figure.

Retention limits 2

- In practice we might model the business and use a stochastic model for future claim rates together with simulation to project the future solvency position of the company.
- We choose retention so that probability of future insolvency is then below a specified (low) figure.
- Other approaches exist, e.g. ask reinsurer!

Managing risk - underwriting

- How can underwriting manage risk?
- The underwriting process
  - Medical and other evidence
  - Interpretation of the evidence
  - Specification of terms
- Group life contracts and "Free cover"

Managing risk

- Protection against anti-selection and over-insurance
- Identification of sub-standard risks
- For substandard risks, identification of most suitable special terms
- Reduction of arbitrage possibilities through adequate risk classification
- Helping to ensure that actual mortality etc experience does not depart too far from that assumed in pricing

Medical and other evidence 1

- Company will obtain evidence
  - for contracts where it is at risk on death or sickness
  - and may also for annuities (longevity risk) if it gives better terms for lives in bad health

Medical and other evidence 2

- Sources of evidence
  - questions on the proposal form
  - reports from applicant’s usual doctor
  - medical examination
  - specialist medical tests
- Apart from the first these cost money $\Rightarrow$ they are only obtained if extent of risk is large enough

Medical and other evidence 3

- In addition to “medical” health, also consider
  - occupation
  - leisure pursuits
  - normal country of residence
  - financial situation (if large sum at risk)

Interpretation of the evidence

- Evidence is looked at by specialist underwriters who convert it into a rating
  - often a percentage of standard mortality etc
  - may also be addition to age
  - or fixed percentage of benefit
- They will make use of
  - doctors employed by the company
  - underwriting manuals prepared by reinsurance companies

Specification of terms

- State of health OK $\Rightarrow$ accept on standard terms
- State of health too bad $\Rightarrow$ decline
- Rest accepted on special terms
  - addition to the normal premium
  - deduction from the normal benefit
  - exclusion clause

Group life – nature of risk

- Special characteristics of risk
  - reduced anti-selection
    - employer decides who joins
    - employer decides level of benefit
  - reduced heterogeneity of risk
    - people at work are usually in good health
- These justify offering “Free cover”

Group life – “Free cover”

- Only if actual benefit exceeds “Free cover” is a life underwritten
- The level of “Free cover” depends on
  - number of members
  - Average size of benefit
  - degree of choice in choosing level of benefit
  - if membership is compulsory or voluntary
  - percentage who join if voluntary
  - if members have to be at work when cover starts
  - competition

Options and guarantees

- Mortality options
- Investment guarantees

Mortality options 1

- Examples
  - option to increase death benefit at standard rates under a term assurance (increase option)
  - option to take out a new term assurance at standard rates when an existing contract expires (renewal option)
  - option to convert a term assurance into an endowment or whole life contract at standard rates (conversion option).

Mortality options 2

- What is the risk?
  - the option is not valuable to those in good health when the option can be taken
  - the option is valuable if the life is not in good health
- Need to price option on basis of
  - an assumed rate of take up of option
  - assumed mortality of those who take it up

Mortality options 3

- What factors affect the risk?
  - the time until the option can be taken - the longer this time the higher the risk
  - the number of chances the policyholder gets to exercise the option - the more chances the higher the risk
  - any conditions attaching to exercising the option (e.g. if all previous options must have been exercised)

Mortality options 4

- What factors affect the risk (contd.)?
  - proportion of eligible lives who exercise the option - the higher this is the lower the risk
  - the extra cost to a policyholder who exercises the option - if the cost is high compared with competitors healthy lives will not take option
  - selective withdrawals before option can be exercised
  - restriction if any on time around option date when option can be exercised

Mortality options 5

- How can we price a simple option?
  - Conventional (British) method
    - assumes every eligible life takes the option
    - assumes that on average they experience Ultimate mortality at the option date
    - unrealistic assumptions BUT simple method
  - “North American experience” method
    - uses assumed proportions taking the option
    - uses assumed mortality of lives taking the option
    - scientific BUT only experience is North American

Investment guarantees - examples

- maturity benefit under an endowment assurance
  - non-unitised contract
  - unitised contract
- guaranteed surrender values under any contract
- option at maturity of an endowment to take an annuity on guaranteed terms

Investment guarantees - Risks 1

- maturity value - non-unitised contract
  - rate of return required
  - investment policy
- maturity value - unitised contract
  - size of the guarantee
  - time until maturity

Investment guarantees - Risks 2

- guaranteed surrender values
  - as for maturity value
  - company does not know when contract will be surrendered
- annuity option
  - cannot invest to meet both the maturity benefit and the annuity payments
  - also will be expense and mortality risks

Investment guarantees - Pricing 1

- maturity value - non-unitised contract
  - contract priced so that on the basis of proposed investment policy there is good possibility that guarantee will be met
- maturity value - unitised contract
  - derive a stochastic investment model appropriate to investments in the unitised fund
  - use simulation to calculate premium that will keep probability of loss below certain figure

Investment guarantees - Pricing 2

- surrender values
  - use simulation as for maturity value of a unitised contract
  - calculate premium required in respect of "worst case" guaranteed surrender value

Investment guarantees - Pricing 3

- annuity option
  - assume company invests to meet maturity value
  - derive a stochastic interest rate model
  - use model to simulate interest rate at maturity
  - for each simulation calculate cost at maturity of taking annuity
  - cost at maturity of guarantee calculated such that probability of making loss is less than a certain figure.

Discontinuance terms

May take the form of

- the payment of a capital sum

or

- conversion of the contract to a paid-up form.

Payment of a capital sum

- Amount may be calculated as
  - earned asset share
  - prospective value
- We need to consider also
  - policyholder expectations
  - new business disclosure and policy conditions
  - frequency of change
  - ease of application

Earned asset share

- On average company will not want to pay more than this
- need in practice to average over time as the earned asset share can fluctuate from day to day
- for with profit contracts may be a fair value to pay

Prospective value

- Difference between prospective value and earned asset share represents profit
  - how much should this profit be?
- For without profit contracts company may want
  - same profit as if contract had continued to end
    - $\Rightarrow$ use realistic assumptions
  - only profit made to date of cancellation
    - $\Rightarrow$ use assumptions closer to premium basis

Prospective value

- For with profit contracts company may want
  - to ensure that value allows for past profit not yet allocated to the contract
  - in practice may use some proportion of a Zillmerised net premium reserve calculated on the premium (technical) basis.

Policyholder expectations

- Near start of contract
  - policyholder may expect to get back "Premiums less expenses"
  - Theoretical value may be negative, but company may accept a loss.
- Near maturity capital sum should be consistent with maturity value.

New business disclosure

- Companies may be required to illustrate values to potential policyholders.
- Legislation may require that values (or method of calculation) is shown in contract conditions.
- Financial press may publish information about the values that companies pay.

Other factors

- Frequency of change in scale of values
  - Scale should not change too often.
- Ease of application
  - It should not be too complicated to calculate the values.

Conversion to a paid-up form

- Benefits should be affordable
  - maximum is earned asset share divided by some “A” factor
  - need to allow for company’s profit
  - this implies using a prospective method
- Benefits should be consistent with maturity values.
- Benefits should be consistent with capital values on cancellation.

Policy alterations

- Limiting conditions and constraints
- Calculation methods
  - equation of policy values
  - SV respread to reduce future premiums
  - PUP plus premium for balance of cover
  - accumulation of premium arrears/surplus
- Underwriting needs

Limiting conditions and constraints 1

- Terms after alteration supportable by earned asset share
- Surrender is limiting case of reduction in term

  $\Rightarrow$ premium after alteration $\rightarrow$ MV – SV as outstanding term $\rightarrow$ 0
- Conversion to paid-up is limiting case of reduction in sum assured

  $\Rightarrow$ premium after alteration $\rightarrow$ 0 as new SA $\rightarrow$ PUP

Limiting conditions and constraints 2

- If benefits increased, terms should be consistent with those for a new contract.
- Methods of calculation should be stable $\rightarrow$ small change in contract terms $\Rightarrow$ small change in premium.
- Terms offered after alteration should avoid option of lapse and re-entry.

Equation of policy values 1

- Value of contract before alteration = value of contract after alteration
- Can be used for any type of alteration, including conversion to paid-up.
- Does it meet the conditions etc.?

Equation of policy values 2

- Terms after alteration supportable by earned asset share

  yes, if value before $\leq$ EAS and basis after not over realistic

- Surrender is limiting case of reduction in term

  yes, if use same basis to calculate SV

- Paid-up is limiting case of reduction in benefit

  yes, if use same basis to calculate PUP value

Equation of policy values 3

- Consistent with terms for new contract if there is increase in benefits

  yes, if use same basis as for new contracts

- Method should be stable

  yes, if use same basis to calculate value before and vale after alteration

- Should avoid option of lapse and re-entry

  this always needs to be checked

SV respread 1

- Calculate premium company would charge for new contract to provide benefits after alteration
- Calculate a “special” surrender value
- Reduce premium by spreading SV over outstanding term
- Simple method to use
- Does it meet the conditions etc.?

SV respread 2

- Terms after alteration supportable by earned asset share

  yes, if special surrender value $\leq$ EAS

- Surrender is limiting case of reduction in term

  yes, automatically

- Paid-up is limiting case of reduction in benefit

  unlikely

SV respread 3

- Consistent with terms for new contract if there is increase in benefits

  yes, automatically

- Method should be stable

  depends very much on relationship between all the bases involved – may be very unstable

- Should avoid option of lapse and re-entry

  yes, automatically

PUP plus premium 1

- Notionally convert contract to paid-up
- Convert paid-up value to correspond with any change in term
- Calculate premium on current premium basis for difference between new benefit required and paid-up value
- Complicated method to use
- Does it meet the conditions etc.?

PUP plus premium 2

- Terms after alteration supportable by earned asset share

  unclear
- Surrender is limiting case of reduction in term

  yes, if SV and PUP value calculated on same bases
- Paid-up is limiting case of reduction in benefit

  yes, automatically

PUP plus premium 3

- Consistent with terms for new contract if there is increase in benefits

  yes, automatically

- Method should be stable

  depends very much on relationship between all the bases involved – may be very unstable

- Should avoid option of lapse and re-entry

  yes, automatically

Accumulation of premium arrears/surplus

- Old premium compared with that the company would have charged if it had issued contract from start for benefits now required.
- Accumulated difference charged/paid to policyholder.
- In practice may only be used for alterations in first year of contract.

Underwriting needs

- No increase in benefit or extension in term

  $\Rightarrow$ no need to consider underwriting
- Increase in benefit

  $\Rightarrow$ apply normal underwriting requirements to increase in benefit, with minimum of statement of good health
- Increase in term

  $\Rightarrow$ usually require statement of good health

Investment – Principles

- Select investments that are appropriate to the nature and term of the liabilities.
- Select investments so as to maximise the overall return on the assets.
- Extent to which a company can depart from first principle to meet the second depends on the extent of its working capital.

First principle

- Absolute matching
  - income from assets exactly matches outgo in respect of liabilities
  - company is not at risk from future changes in investment conditions
  - in practice requires that net liability outgo is always positive, which is not usually true.
- Immunisation

Skip mathematics

Immunisation – Mathematics 1

Define

A_t = asset proceeds at time t

L_t = liability outgo at time t

δ = ruling force of interest at time $t = 0$

$$VA = \int_0^\infty A_t e^{-\delta t} dt$$

$$VL = \int_0^\infty L_t e^{-\delta t} dt$$

Immunisation – Mathematics 2

Assume that at present time $VA = VL$

and that force of interest changes $\delta \to \delta + \varepsilon$

so that $VA \to VA'$

and $VL \to VL'$

$$VA' - VL' = (VA - VL) + \varepsilon \frac{d(VA - VL)}{d\delta} + \frac{\varepsilon^2}{2!} \frac{d^2(VA - VL)}{d\delta^2} + \ldots$$

Immunisation – Mathematics 3

- For small changes in the force of interest we can ignore higher order terms
- Company will not make a loss if second term is zero and third term is not negative

$$\frac{d(VA-VL)}{d\delta} = 0$$

$$\frac{d^2(VA-VL)}{d\delta^2} \geq 0$$

Immunisation – Mathematics 4

- Expanding the above differentials we obtain the two conditions to the right

$$\int_0^\infty (A_t - L_t)\, t\, e^{-\delta t} dt = 0$$

$$\int_0^\infty (A_t - L_t)\, t^2 e^{-\delta t} dt \geq 0$$

Immunisation

- The mean discounted term of the asset proceeds equals the mean discounted term of the liability outgoes.
- The spread about the mean of the discounted term of the asset proceeds is greater than the spread about the mean of the discounted term of the liability outgoes.

Immunisation - problems

- Theory is only valid for small changes in the force of interest.
- It assumes that the same force of interest applies to all assets.
- It requires a company continually to rearrange its investments.
- Theory does not allow for company shares etc. and for with profit contracts.

Second principle

- Maximising investment return will
  - improve the use of the company's working capital
  - help to increase new business and hence profits
  - help to increase bonuses to policyholders which will then increase new business and profit.

Third principle

- The problem with investing in company shares is that they can go down in value significantly.
- The existence of working capital provides a “reserve” to cover this risk.

Asset-liability matching

- Choice of investments is affected by
  - nature and term of the liabilities
  - extent of the working capital
  - supervisory framework
- Supervisors may impose particular matching requirements or restrict the range of assets that the company can invest in.

Nature and term of liabilities 1

- Liability outgo/income consists of
  - benefit payments
  - expense payments
  - premiums
- Benefit payments can be
  - guaranteed
  - non-guaranteed
  - index-linked
- Expenses are more or less index-linked
- Premiums are guaranteed

Nature and term of liabilities 2

- Liability outgo/income can then be split into
  - guaranteed
  - discretionary
  - index-linked
- Guaranteed $\rightarrow$ invest in guaranteed securities (using asset-liability modelling to get best match)
- Discretionary $\rightarrow$ invest in assets producing highest expected return
- Index-linked $\rightarrow$ invest in index-linked stocks

Working capital

- Existence of working capital allows company to depart from matching strategies.
- It provides a reserve against adverse changes in capital values.
- The use of such capital however has an associated cost – shareholders need to be paid for it.
- The optimal amount of capital required can be assessed using asset-liability modelling.

Asset-liability modelling

- Assume assets are invested according to a certain investment policy and company has a certain level of working capital.
- Project the assets and liabilities (maybe using a model) using a set of assumptions.
- Either use a number of sets of assumptions or stochastic assumptions and simulation.
- We can use this
  - to assess the solvency position in each future year
  - to investigate the effect on shareholder profits ...

Stochastic investment models

- Much has been written on these in recent years.
- In the UK the following ARIMA type models are often used.

$$\ln Y(t) \quad = \quad \mu_y + \alpha(\ln Y(t-1) - \mu_y) + \eta_t$$

where $Y(t)$ = interest rate at time t

μ_y = assumed average "force" of interest rate

α = auto-regressive parameter

η_t = normally distributed r. v. with zero mean

Profit distribution and risk

- Margins for future adverse experience
- Business objectives of the company
- Policyholder expectations
- Provision of capital

Margins for future adverse experience

- Existence of profit distribution implies that the premiums contain margins (implicit or explicit).
- We can also view these margins as margins against adverse future experience.
- How far we can view them this way may depend on policyholder expectations.

Business objectives of the company

- One objective of company is likely to be to maximise the profit distribution to policyholders.
- Aim of this is to improve the company's competitive position and hence get more new business.
- Actuary should keep this objective in mind when advising on the profit to distribute.

Policyholder expectations

- Policyholders may have expectations as regards the level of profit and the method of distribution.
- These expectations may come from
  - company documentation
  - company's past practice
  - general practice in the market
- If fail to meet these expectations, will have dissatisfied policyholders and less new business.

Provision of capital 1

- Premium rates contain margins that generate profit.
- Pace at which profits arise and pace at which they are distributed may not be the same.
- If part of profit is deferred to the future, it will increase the company’s working capital.

Provision of capital 2

- The increase in working capital allows the company to take on greater risk.
- In particular it may allow the company to invest more freely and hence more profitably.
- Working capital can also be used to meet financing strains under new business.

Provision of capital 3

- Where profit is not distributed as it arises, there will be years when more is distributed than has arisen in that year.
- Over time the two should balance out.
- If not the company can have problems – consistent under-distribution is as bad as consistent over-distribution.
- Extent of deferral of profit depends on profit distribution system.

Provision of capital 4

- Additions to benefits
  - method has greatest degree of flexibility
  - the more profit is given in terminal bonus form the greater is the deferral
  - super-compound RB defers more than compound which defers more than simple
  - as used in the UK the method can lead to a significant deferral of profit .

Provision of capital 5

- European methods
  - it is usual to distribute all the profit as it arises
  - investment is usually in Government stocks and hence working capital is not in any case required for this purpose.

Provision of capital 6

- Contribution method
  - sometimes with this method companies do not distribute all the profit as it arises, but give a terminal dividend in addition to the regular ones
  - extent of any deferral of profit is usually less than under the additions to benefits method.

Capital requirement 1

- Why does a company need capital?
  - to cover any investment mismatch
  - to finance new projects – new administration system, new product line etc.
  - to finance the writing of new business – the marginal cost of issuing a new contract
  - to finance the purchase of another company

Capital requirement 2

- Where is this capital?
  - Working capital is excess of value of assets over value of liabilities
  - the amount continually changes –reduces as new projects arise etc. and increases as profits emerge from those projects etc.
  - if the capital becomes negative, company is insolvent $\Rightarrow$ capital must be managed.

Capital requirement 3

- Managing the capital
  - To manage the capital we can use cash-flow projections of the company’s business
  - these will show if the company can achieve its business plans and still remain solvent
  - nature of the projections is the same as those described for investigating investment strategies.

Capital requirement 4

- Other sources of capital
  - shareholders can put new money into the company – if it will achieve an adequate rate of return
  - the company can make use of reinsurance arrangements
  - the company may be able to securitise part of its embedded value.

Monitoring experience

- <u>Reasons for monitoring experience</u>
- <u>Data required</u>
- <u>Analysis of experience</u>
- <u>Analysis of surplus</u>
- <u>Analysis of change in embedded value</u>
- <u>Using the results</u>

Reasons for monitoring experience

- to develop earned asset shares
- to update assumptions as to future experience
- to monitor any adverse trends in experience so as to take corrective action
- to provide management information.

Data required

- Must be a reasonable volume of stable, consistent data from which we can deduce future experience and trends.
- Data should be divided into sufficiently homogeneous risk groups, according to relevant risk factors.
- BUT each data cell must have a credible amount of data.
- Hence will usually be a compromise between the two.

A of E – Mortality etc.

- Analyse by
  - type of contract
  - age
  - sex
  - duration from entry
  - degree of underwriting
  - source of business
  - smoker/non-smoker status
  - cause of claim (critical illness)

A of E – Withdrawals 1

Analyse by (in order of importance)

- type of contract
- duration from entry
- source of business
- others
  - frequency and size of premium
  - premium payment method
  - original term of contract
  - sex and age

A of E – Withdrawals 2

- Withdrawal rates in future are also affected by
  - current economic situation
  - competitive position of the contract
- These are not usually allowed for in an analysis of past experience.

A of E – Expenses 1

- Commission does not need to be investigated.
- We can divide other expenses in theory into
  - *direct* – those that depend on volume of new or existing business
  - *overheads* – the rest
- In practice there is not a clear dividing line between the two.

A of E – Expenses 2

- For purposes of analysis divide into
  - initial expenses
  - renewal expenses
  - termination expenses
  - investment expenses
- First three can be further split into those
  - Proportional to number of contracts (majority)
  - Proportional to amount of benefit (underwriting)
  - Proportional to the amount of premium (marketing)

A of E – Expenses 3

- Staff costs most important
  - divide staff into 3 groups
    - those whose work fits in one cell only → allocate costs directly
    - those whose work fits in more than one cell → allocate costs according to time-sheets
    - others → allocate costs pragmatically
- Investment costs netted from investment income

A of E – Expenses 4

- Computer costs can be split according to time spent on different tasks.
- Building costs can be allocated to department salaries by floor space occupied.
- One-off capital costs should be amortised over their useful lifetime.

Overheads

- Big problem
- No correct way of dealing with them
  - if load too much - not competitive
  - if too little - may become insolvent
- Spread over expected number of contracts
  - need assumptions about future new business
- Marginal costing may be used

Investment earnings

- Earnings affected by
  - spread between sectors
  - company's investment policy
- Consider in analysis
  - current earnings by asset type
  - expected reinvestment returns
  - likely future spread of investments by type

Analysis of surplus

- A company will want to do this
  - to show financial effect of divergences between valuation assumptions and actual experience
  - to provide a check on valuation process and data
  - to identify non-recurring items of profit
  - to give information on trends in experience

Example

Analysis of change in EV

- A company may do this in order to
  - validate the calculations, assumptions and data used
  - reconcile the values for successive years
  - provide management information
  - provide detailed information for publication in the company's accounts, e.g. the value of new business.

Example

Using the results

- Results of monitoring the experience will be used by the actuary to reassess her or his view of the future.
- This may result in changes to models used for pricing or reserving or to the assumptions used in them.
- Process is iterative as illustrated in the Actuarial Control Cycle

Actuarial Control Cycle